PHY 312: Relativity and Cosmology, Spring 2016

Problem Set 10

Distribution: Tue Apr 12, 2016; Due: Tue Apr 26, 2016

1 Cosmological Distances

In class we have seen that we can calculate the comoving distance \( d_C(z) \) as a function of redshift as

\[
d_c(z) = \int_0^z \frac{dz}{H_0 \sqrt{\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda}}
\]  

(1)

Here \( \Omega_M \) is the matter-dominated energy density, \( \Omega_\Lambda \) is the vacuum energy density, and \( \Omega_K = 1 - \Omega_M - \Omega_\Lambda \) is the curvature energy density. They are all normalized by the critical energy density \( \rho_c \), i.e. \( \Omega_M = \rho_M/\rho_c \), etc. The Hubble constant is about \( H_0 = 68 \text{ km/sec/Mpc} \), or \( 1/H_0 = 4.4 \text{ Gigaparsec} \).

We can find the angular comoving distance \( D(z) \) as

\[
D(z) = d_c(z) \quad \text{if} \quad \Omega_K = 0
\]

(2)

\[
D(z) = \frac{\sinh [d_c(z)H_0\sqrt{\Omega_K}]}{H_0\sqrt{\Omega_K}} \quad \text{if} \quad \Omega_K > 0
\]

(3)

\[
D(z) = \frac{\sin [d_c(z)H_0\sqrt{-\Omega_K}]}{H_0\sqrt{-\Omega_K}} \quad \text{if} \quad \Omega_K < 0
\]

(4)

Finally, angular diameter distance \( D_A(z) \) and luminosity distance \( D_L(z) \) are given by

\[
D_A(z) = \frac{D(z)}{1+z}
\]

(5)

\[
D_L(z) = D(z)(1+z)
\]

(6)

Calculate the luminosity distance vs. redshift relation \( D_L(z) \) for the following 3 universes:

a) \( \Omega_M = 1.0, \Omega_\Lambda = 0.0, \Omega_K = 0.0 \)

b) \( \Omega_M = 0.3, \Omega_\Lambda = 0.0, \Omega_K = 0.7 \)

c) \( \Omega_M = 0.3, \Omega_\Lambda = 0.7, \Omega_K = 0.0 \)

This is the \( \Lambda \)-CDM model that fits the observational data best.

Use your favorite software tool to numerically evaluate the integrals for a range of \( z = 0.1...10 \).