1 Event separation

Show that the interval between two events \((t_1, x_1, y_1, z_1)\) and \((t_2, x_2, y_2, z_2)\), defined by

\[
s^2 = -(t_2 - t_1)^2 + (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2
\]

(1)
is invariant under Lorentz transformation. What does \(s^2\) become if \(t_2 = t_1\)?

Convention:

- If \(s^2 > 0\), we quote the value of \(s\) for the space-time interval, and call it space-like.
- If instead \(s^2 < 0\), we define \(\tau^2 = -s^2\), quote the value of \(\tau\) for the space-time interval, and call it time-like.

2 Spacetime map

a) Space-time map of some events.

The laboratory space and time measurements of events 1 through 5 are plotted in the figure. Compute the value of the spacetime interval:

- between event 1 and event 2.
- between event 1 and event 3.
- between event 1 and event 4.
- between event 1 and event 5.
- A rocket moves with constant velocity from event 1 to event 2. That is, events 1 and 2 occur at the same place in this rocket frame. What time lapse is recorded on the rocket clock between these two events?

Note: Follow the convention from problem 1 for reporting space-like and time-like separations.

b) Define the reference frame for the rocket from a) in two steps:

1) First perform a translation such that event 1 becomes the origin.
2) Next perform a Lorentz boost such that the new space coordinate for events 1 & 2 are 0.

Express the position of all event in this new frame. Verify that the space-time intervals from part a) are unchanged.
3 Time stretching with $\mu$-mesons

At heights of 10 to 60 kilometers above Earth, cosmic rays continually strike nuclei of oxygen and nitrogen atoms and produce muons ($\mu$-mesons: elementary particles of mass equal to 207 electron masses produced in some nuclear reactions). Some of the muons move vertically downward with a speed nearly that of light. Follow one of the muons on its way down. In a given sample of muons, half of them decay to other elementary particles in 1.5 microseconds ($1.5 \times 10^{-6}$ seconds), measured with respect to a reference frame in which they are at rest. Half of the remainder decay in the next 1.5 microseconds, and so on. Analyze the results of this decay as observed in two different frames. Idealize the rather complicated actual experiment to the following roughly equivalent situation: All the muons are produced at the same height (60 kilometers); all have the same speed; all travel straight down; none are lost to collisions with air molecules on the way down.

- **a** Approximately how long a time will it take these muons to reach the surface of Earth, as measured in the Earth frame?

- **b** If the decay time were the same for Earth observers as for an observer traveling with the muons, approximately how many half-lives would have passed? Therefore what fraction of those created at a height of 60 kilometers would remain when they reached sea level on Earth? You may express your answer as a power of the fraction $1/2$.

- **c** An experiment determines that the fraction $1/8$ of the muons reaches sea level. Call the rest frame of the muons the rocket frame. In this rocket frame, how many half-lives have passed between creation of a given muon and its arrival as a survivor at sea level?

- **d** In the rocket frame, what is the space separation between birth of a survivor muon and its arrival at the surface of Earth? (Careful!)

- **e** From the rocket space and time separations, find the value of the spacetime interval between the birth event and the arrival event for a single surviving muon.


Problems taken from *Spacetime Physics* Edwin F. Taylor, John Archibald Wheeler