A brief history of the universe

We are now ready to play back what happened in the universe so far:

**Redshift**:
- $z = 0$ : Now
- $z = 1-2$ : Formation of Galaxy clusters
- $z = 5-7$ : First observable Galaxies
  - First quasars (annihilating supermassive Black Holes)
- $z \approx 6-20$ : Reionization: Light from first first collapsed objects ionized the neutral Hydrogen

$z = 15-40$ : First collapsed objects

$z \approx 40-1100$ : Dark ages: No significant starlight, all Hydrogen neutral

$z = 1100$; $T = 4$ eV: Photon decoupling ("Recombination")
- $T \approx 3000$ K \( \Rightarrow \) \( e^- + p \rightarrow \# \) (neutral hydrogen)
- Photons \( p \) don't interact anymore after this
- Lost surface of scattering for \( p \)
- Cosmic Microwave Background

(CMB) is a snapshot from \( z = 1100 \)
\[ z = 6000 \quad T = 1.40 \times 10^4 K \quad \text{Radiation-Matter equality} \]

\[ \frac{\Omega_{\text{rad}}}{\Omega_{\text{matter}}} = \frac{aT^4}{\rho_{\text{matter}}} = \frac{\Omega_{\text{rad}}}{\Omega_{\text{matter}}} \frac{T^4}{(1+z)^3 + 5/6} \]

\[ \Rightarrow \text{for} \quad z \leq 6000 \quad \text{matter dominated} \]

\[ z \geq 6000 \quad \text{radiation dominated} \]

\[ z = 6.6 \times 10^8 \quad T \sim 0.15 \text{MeV} \sim 1.7 \times 10^9 K \]

- Big Bang Nucleosynthesis: \(D, ^3\text{He}, ^4\text{He}, ^7\text{Li}\)
  - form in the current observable today

\[ e^+e^- \text{ annihilation} \]

\[ (T \approx 1 \text{MeV}) \quad e^+e^- \rightarrow 2\gamma \]

\[ z = 4.4 \times 10^9 \quad T \sim 1 \text{MeV} \sim 1.2 \times 10^{10} K \]

- Neutrinos \& decouple from other matter

\[ z = 8.6 \times 10^{11} \quad T \sim 150 \text{MeV} \sim 1.7 \times 10^{12} K \]

- Quark-gluon plasma to Hadron transition

\[ z = 9 \times 10^{14} \quad T \sim 200 \text{GeV} \sim 2 \times 10^{15} K \]

- Electroweak transition; \(W^+\) & \(Z\) bosons decouple
Electromagnetic interactions:

Weak interaction:
$2 \lesssim 10^{15}$ No experimental data...
$\gtrsim 2 \times 10^{15} \text{K} \approx 200 \text{GeV}$

\[ z \]

Ideas:
- Grand unification: $T \approx 10^{16} \text{GeV}$
- Inflation: $T \approx 10^{16} \text{GeV}$

Planck scale: $T = m_{\text{Pl}} = \sqrt{\frac{\hbar c^5}{8\pi G}} \approx 10^{19} \text{GeV}$
Hydrogen ionization/recombination reaction

\[ H + e^- \leftrightarrow e^- + p \]

\( E = 13.6 \text{eV} \)

\( = \) ionization Energy
\( \text{for hydrogen} \)

At high enough temperature: plenty of high energy photons:

\( \Rightarrow \) all Hydrogen is ionized!

\( \Rightarrow \) Lots of free charges around: \( \Rightarrow \) plasma!

\( \Rightarrow \) photons keep scattering
\( \Rightarrow \) universe is opaque

For \( T \) small enough:

\( \Rightarrow p \) no longer able to ionize \( H \) \( \Rightarrow \) Hydrogen forms

\( \Rightarrow \) universe is transparent!
Recombination happens $@ z \approx 1100, T = \frac{1}{3} eV \approx 3000K$

Note: ionization energy $E_i = 13.6 eV \gg T$

but: many more photons than protons than photons

$$H + p \leftrightarrow e^- + p$$

only shift to the left $@ \approx 3000K$
So why is the recombination interesting?

⇒ For the first time the universe becomes transparent to light.
⇒ those photons last scattered at \( z = 1100 \), i.e. the universe was \( \times 1000 \) smaller.
⇒ This is the oldest observable light.

⇒ At the time of last scattering those photons were in thermal equilibrium with their environment.
⇒ Since then they propagated freely and only underwent red-shifting.

⇒ Today we can observe them as microwave radiation with a temperature of \( T = 2.725 \, \text{K} \), coming from every direction.
⇒ The Cosmic Microwave Background (CMB).

⇒ It is extremely isotropic:
  - Largest deviation: Dipole due to Doppler shift from our own "pervious" motion
    \[
    \frac{\Delta T}{T} = 1.2 \times 10^{-3}
    \]
    from \( v = \frac{368 \pm 2}{300} \) sec
  - After that: fluctuations of the size \( \frac{\Delta T}{T} \leq 10^{-5} \) are visible.
The universe has the same $T$ in every direction ($\text{to} \leq \frac{\epsilon T}{10^{-6}}$)

Why? Was everything in a thermal equilibrium?

Question: Can patches in different directions be causally connected?

Let's assume a radiation-dominated universe (of charged particles), then

$$ds^2 = -dt^2 + a^2(t) dr^2$$

How far (in comoving coordinates $r$) can light travel since the Big Bang?

$$H^2 = \frac{\dot{a}^2}{a^2} = H_0^2 \cdot \frac{1}{a^4} \Rightarrow \text{radiation dominated}$$

$$\Rightarrow \dot{a}^2 = H_0^2 \cdot \frac{1}{a^2}$$

$$\Rightarrow a = H_0 \cdot \frac{1}{a} \Rightarrow da = H_0 dt$$

$$\Rightarrow \int da = \int H_0 dt \Rightarrow \frac{1}{2} a^2 = H_0 t \Rightarrow t = \frac{a^2}{2H_0}$$

$$\Rightarrow \dot{a} = \sqrt{2H_0} \frac{a}{t}$$

$$t_{now} = \frac{1}{2H_0}$$
Now, \[ d\tau = \frac{a}{r} \, dr \]
\[ \Rightarrow \int_0^r \frac{d\tau}{a} = \int_0^r \frac{dr}{r} = \ln r \]
\[ \Rightarrow r = \int_0^r \frac{e^{\ln r}}{(2H_0)^2} \, dr = \sqrt{\frac{2}{H_0^2}} \, t^{\frac{1}{2}} \]
\[ t = \frac{a}{H_0} \]

i.e. at the CMB scatter surface, \( a = \frac{1}{1000} \)
\[ \Rightarrow \text{at past light travelled } r = \frac{1}{H_0 \cdot 1000} \]

\[ r = 0 \Rightarrow \frac{1}{1000} \text{ max causally connected patch} \]

\[ \Rightarrow B = \frac{1}{1000} \text{ rad} = \frac{180^\circ}{\pi} \cdot \frac{1}{1000} \approx 0.06^\circ \]

Yet all of the CMB is \( \approx \) at the same temperature.
Can we fix this?

What if \( S = S_0 \text{ const} \)?

\[
a^* = H_0 \ a
\]

\[
\frac{da}{a} = H_0 \, dt
\]

\[
en a = H_0 \, t
\]

\[
a = a_0 \cdot e^{H_0 \, t} \quad \text{a + always}
\]

\[
f = \int_{-\infty}^{+\infty} \frac{dt}{a} = \frac{1}{H_0} \left[ \frac{e^{-H_0 t}}{H_0 a_0} \right]_{-\infty}^{+\infty} = \infty
\]

⇒ If we had a cosmological constant-like evolution in the past, i.e.

\[
a \propto e^{H_0 t} \iff \text{inflation}
\]

⇒ It is natural that all observable points in the sky once were in thermal equilibrium.